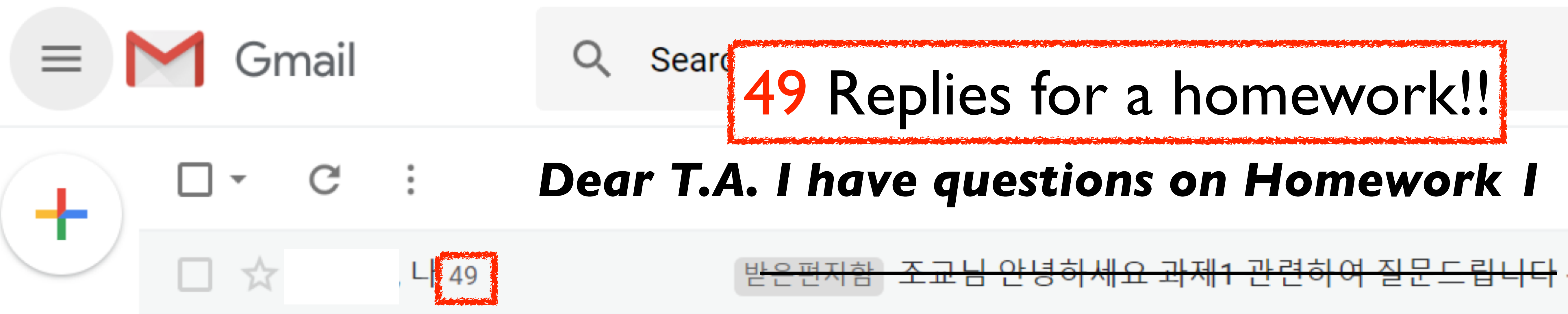
Automatic Diagnosis and Correction of Logical Errors for Functional Programming Assignments

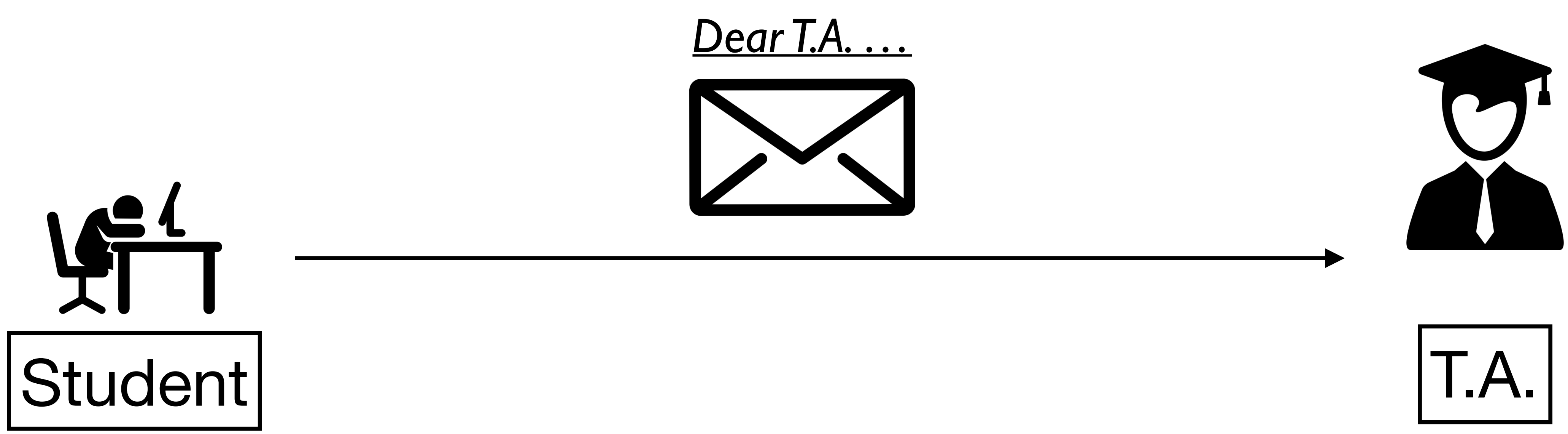
Junho Lee, Dowon Song, Sunbeom So, and Hakjoo Oh
Korea University

9 November 2018
OOPSLA`18 @ Boston, U.S.A.

Motivation

- T.A. experience in functional programming course.
- A lot of e-mails about assignments





Motivation

Student's implementation:

```
type aexp =
   |CONST of int
   | VAR of string
   | POWER of string * int
   | TIMES of aexp list
   | SUM of aexp list

type env = (string * int * int) list

let diff : aexp * string -> aexp
= fun (aexp, x) ->

  let rec deployEnv : env -> int -> aexp list
  = fun env flag ->
  match env with
  | hd::tl ->
  (
   match hd with
   |(x, c, p) ->
     if (flag = 0 && c = 0) then deployEnv tl flag
     else if (x = "const" && flag = 1 && c = 1) then deployEnv tl flag
     else if (p = 0) then (CONST c)::(deployEnv tl flag)
     else if (c = 1 && p = 1) then (VAR x)::(deployEnv tl flag)
     else if (p = 1) then TIMES[CONST c; VAR x]::(deployEnv tl flag)
     else if (c = 1) then POWER(x, p)::(deployEnv tl flag)
     else TIMES [CONST c; POWER(x, p)]::(deployEnv tl flag)
)
| [] -> []
in

let rec updateEnv : (string * int * int) -> env -> int -> env
= fun elem env flag ->
match env with
| (hd::tl) ->
   (
   match hd with
   | (x, c, p) ->
     (
     match elem with
     |(x2, c2, p2) ->
       if (flag = 0) then
         if (x = x2 && p = p2) then (x, (c + c2), p)::tl
         else hd::(updateEnv elem tl flag)
       else
         if (x = x2) then (x, (c*c2), (p + p2))::tl
         else hd::(updateEnv elem tl flag)
     )
   )
| [] -> elem::[]
in

let rec doDiff : aexp * string -> aexp
= fun (aexp, x) ->
match aexp with
| CONST _ -> CONST 0
| VAR v ->
   if (x = v) then CONST 1
   else CONST 0
| POWER (v, p) ->
   if (p = 0) then CONST 0
   else if (x = v) then TIMES ((CONST p)::POWER (v, p-1)::[])
   else CONST 0
| TIMES lst ->
   (
     match lst with
       (
         match (hd, diff_hd, tl, diff_tl) with
         | (CONST p, CONST s, [CONST r], CONST q) -> CONST (p*q + r*s)
         | (CONST p, _, _, CONST q) ->
           if (diff_hd = CONST 0 || tl = [CONST 0]) then CONST (p*q)
           else SUM [CONST(p*q); TIMES(diff_hd::tl)]
         | (_, CONST s, [CONST r], _) ->
           if (hd = CONST 0 || diff_tl = CONST 0) then CONST (r*s)
           else  SUM [TIMES [hd; diff_tl]; CONST(r*s)]
         | _ ->
           if (hd = CONST 0 || diff_tl = CONST 0) then TIMES(diff_hd::tl)
           else if (tl = [CONST 0] || diff_hd = CONST 0) then TIMES [hd; diff_tl]
           else SUM [TIMES [hd; diff_tl]; TIMES (diff_hd::tl)]
       )
     | [] -> CONST 0
   )
| SUM lst -> SUM(List.map (fun aexp -> doDiff(aexp, x)) lst)
in

let rec simplify : aexp -> env -> int -> aexp list
= fun aexp env flag ->
match aexp with
| SUM lst ->
   (
     match lst with
     | (CONST c)::tl -> simplify (SUM tl) (updateEnv ("const", c, 0) env 0) 0
     | (VAR x)::tl -> simplify (SUM tl) (updateEnv (x, 1, 1) env 0) 0
     | (POWER (x, p))::tl -> simplify (SUM tl) (updateEnv (x, 1, p) env 0) 0
     | (SUM lst)::tl -> simplify (SUM (List.append lst tl)) env 0
     | (TIMES lst)::tl ->
       (
         let l = simplify (TIMES lst) [] 1 in
         match l with
         | h::t ->
           if (t = []) then List.append l (simplify (SUM tl) env 0)
           else List.append (TIMES l::[]) (simplify (SUM tl) env 0)
         | [] -> []
       )
     | [] -> deployEnv env 0
   )
| TIMES lst ->
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     match lst with
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         match l with
         | h::t ->
           if (t = []) then List.append l (simplify (TIMES tl) env 1)
            else List.append (SUM l::[]) (simplify (TIMES tl) env 1)
          | [] -> []
        )
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      | [] -> deployEnv env 1
   )
in

let result = doDiff (aexp, x) in
match result with
| SUM _ -> SUM (simplify result [] 0)
| TIMES _ -> TIMES (simplify result [] 1)
| _ -> result
```

Solution:

```
let rec diff : aexp * string -> aexp
= fun (e, x) ->
  match e with
  | Const n -> Const 0
  | Var a -> if (a <> x) then Const 0 else Const 1
  | Power (a, n) -> if (a <> x) then Const 0 else Times [Const n; Power (a, n-1)]
  | Times l ->
    begin
    match l with
    | [] -> Const 0
    | hd::tl -> Sum [Times ((diff (hd, x))::tl); Times [hd; diff (Times tl, x)]]
    end
  | Sum l -> Sum (List.map (fun e -> diff (e,x)) l)
```

TA:
Hard to generate feedback!

Students:
Solution is meaningless…

Goal

Student's implementation:

```
type aexp =
   |CONST of int
   | VAR of string
   | POWER of string * int
   | TIMES of aexp list
   | SUM of aexp list

type env = (string * int * int) list

let diff : aexp * string -> aexp
= fun (aexp, x) ->

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       if (flag = 0) then
         if (x = x2 && p = p2) then (x, (c + c2), p)::tl
         else hd::(updateEnv elem tl flag)
       else
         if (x = x2) then (x, (c*c2), (p + p2))::tl
         else hd::(updateEnv elem tl flag)
     )
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 let rec doDiff : aexp * string -> aexp
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   else if (x = v) then TIMES ((CONST p)::POWER (v, p-1)::[])
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 | TIMES lst ->
   (
     match lst with
       (
       match (hd, diff_hd, tl, diff_tl) with
       | (CONST p, CONST s, [CONST r], CONST q) -> CONST (p*q + r*s)
       | (CONST p, _, _, CONST q) ->
         if (diff_hd = CONST 0 || tl = [CONST 0]) then CONST (p*q)
         else SUM [CONST(p*q); TIMES(diff_hd::tl)]
       | (_, CONST s, [CONST r], _) ->
         if (hd = CONST 0 || diff_tl = CONST 0) then CONST (r*s)
         else SUM [TIMES [hd; diff_tl]; CONST(r*s)]
       | _ ->
         if (hd = CONST 0 || diff_tl = CONST 0) then TIMES(diff_hd::tl)
         else if (tl = [CONST 0] || diff_hd = CONST 0) then TIMES [hd; diff_tl]
         else SUM [TIMES [hd; diff_tl]; TIMES (diff_hd::tl)]
       )
     | [] -> CONST 0
   )
 | SUM lst -> SUM(List.map (fun aexp -> doDiff(aexp, x)) lst)
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 let rec simplify : aexp -> env -> int -> aexp list
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     | (CONST c)::tl -> simplify (SUM tl) (updateEnv ("const", c, 0) env 0) 0
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     | (SUM lst)::tl -> simplify (SUM (List.append lst tl)) env 0
     | (TIMES lst)::tl ->
       (
         let l = simplify (TIMES lst) [] 1 in
         match l with
         | h::t ->
           if (t = []) then List.append l (simplify (SUM tl) env 0)
           else List.append (TIMES l::[]) (simplify (SUM tl) env 0)
         | [] -> []
       )
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     | (POWER (x, p))::tl -> simplify (TIMES tl) (updateEnv (x, 1, p) env 1) 1
     | (SUM lst)::tl ->
       (
         let l = simplify (SUM
         match l with
         | h::t ->
           if (t = []) then Li
            else List.append (S
         | [] -> []
       )
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   )
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 let result = doDiff (aexp, x) in
 match result with
 | SUM _ -> SUM (simplify result [] 0)
 | TIMES _ -> TIMES (simplify result [] 1)
 | _ -> result
```

Time: 3.4 sec

Solution:

```
let rec diff : aexp * string -> aexp
= fun (e, x) ->
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  | Power (a, n) -> if (a <> x) then Const 0 else Times [Const n; Power (a, n-1)]
  | Times l ->
    begin
    match l with
    | [] -> Const 0
    | hd::tl -> Sum [Times ((diff (hd, x))::tl); Times [hd; diff (Times tl, x)]]
    end
  | Sum l -> Sum (List.map (fun e -> diff (e,x)) l)
```
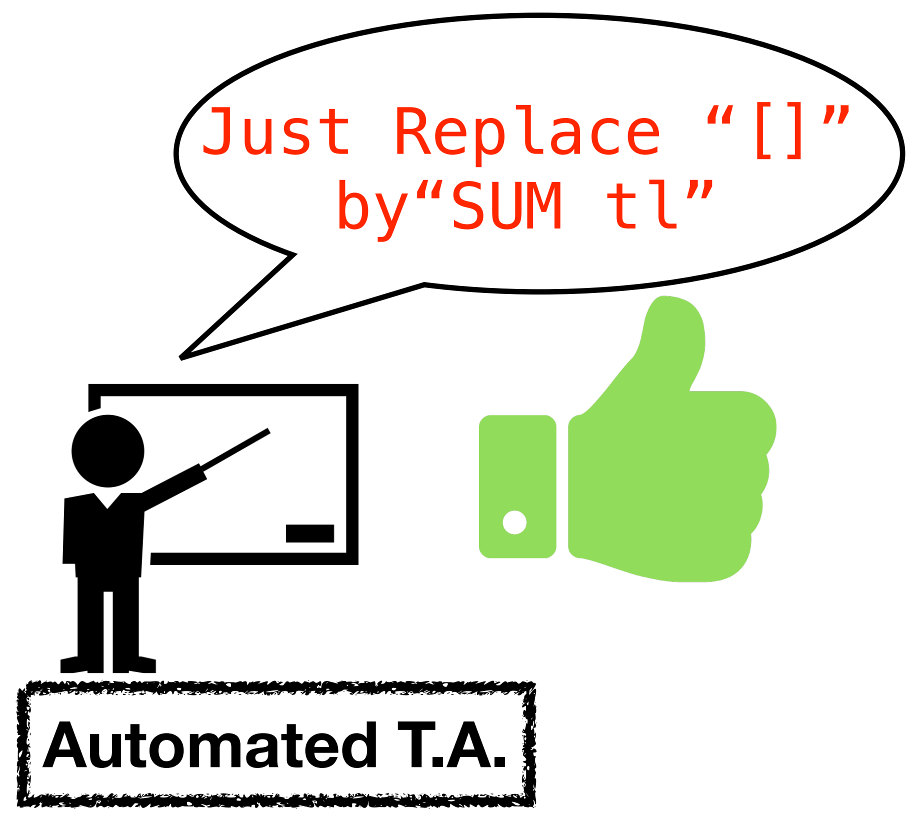
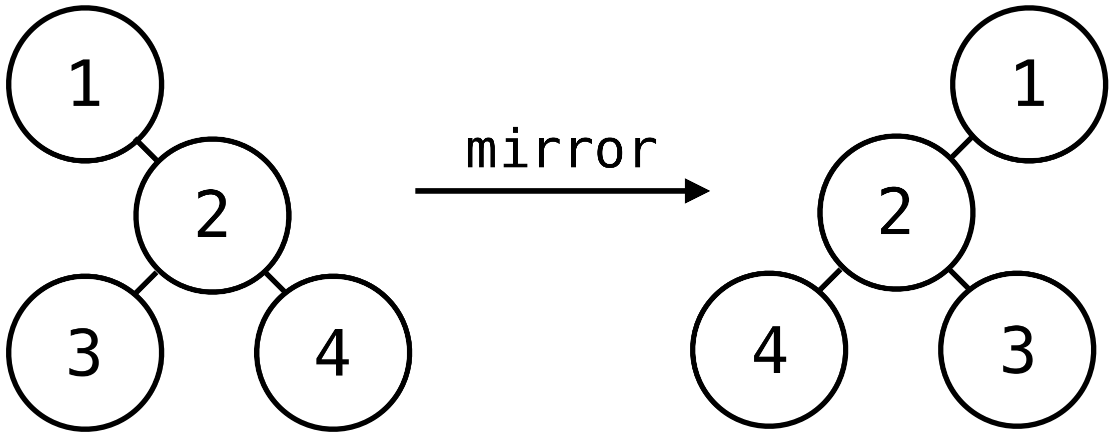



Example 1: Mirroring Tree

- Warming up!

```
type btree =
  | Empty
  | Node of int * btree * btree

let rec mirror tree =
  match tree with
  | Empty -> Empty
  | Node (n,l,r) -> Node (n,r,l)
```



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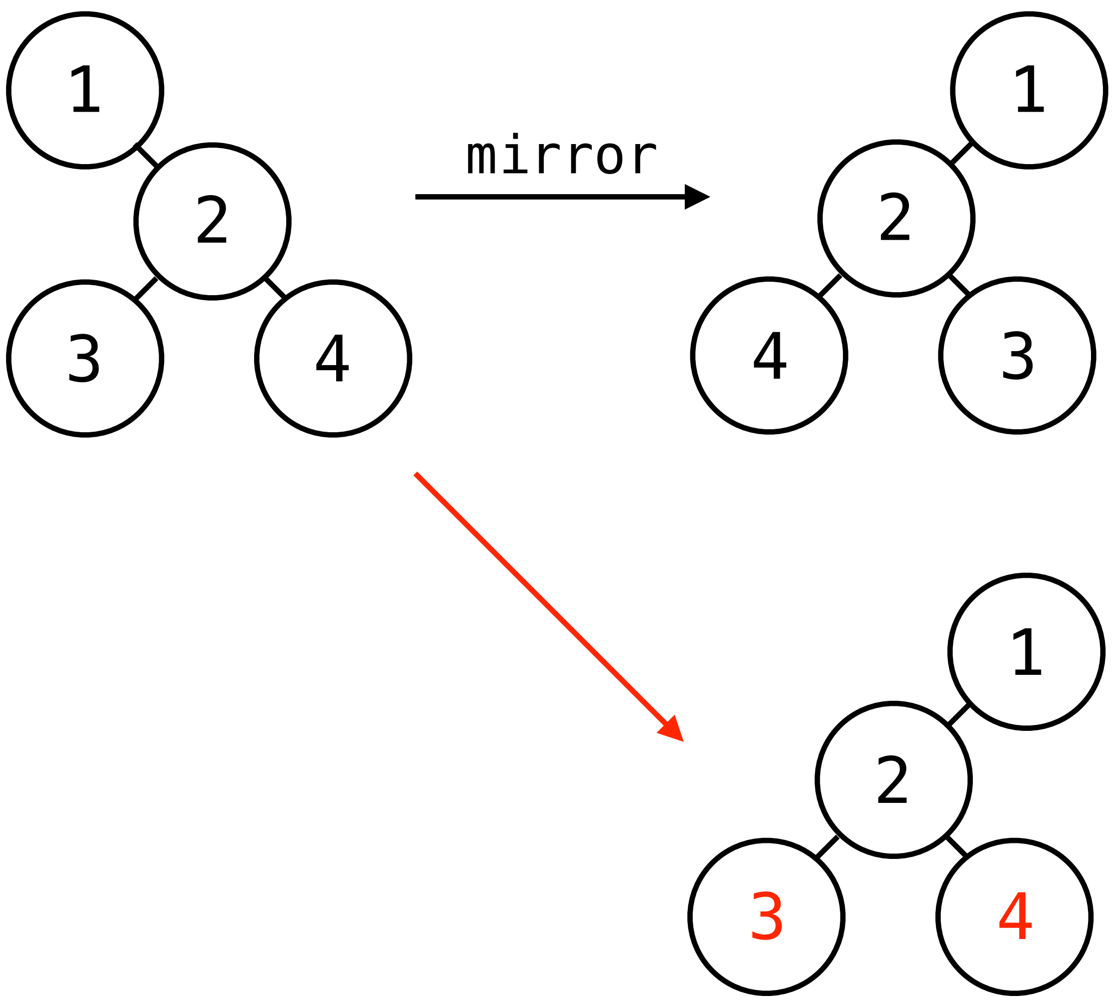

Example1: Mirroring Tree

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type btree =
  | Empty
  | Node of int * btree * btree

let rec mirror tree =
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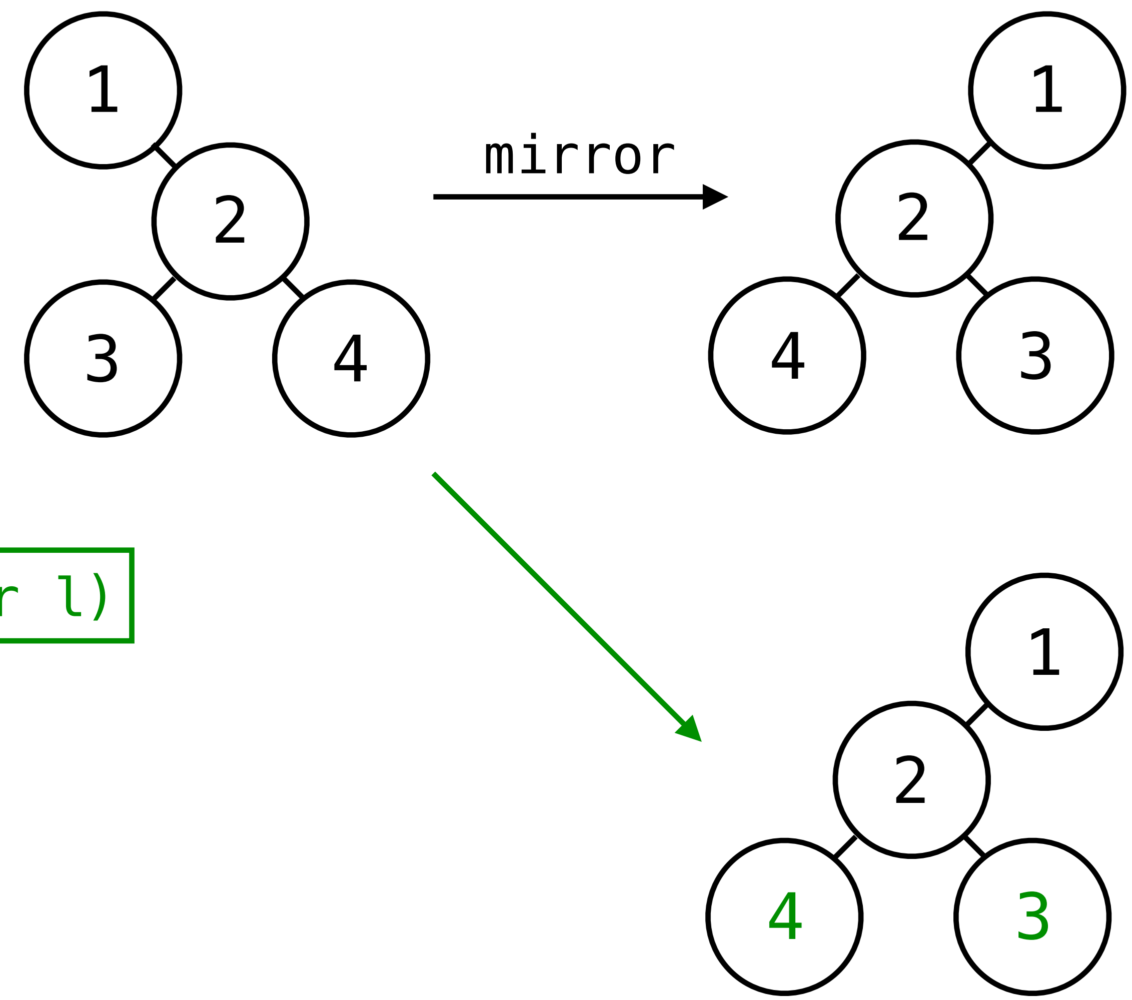
```

FixML: Node (n, mirror r, mirror l)

Time: 0.1 sec



Example2: Natural Numbers

- More complicated program

```
type nat =
  |ZERO
  |SUCC of nat

let rec natadd n1 n2 =
  match n1 with
  |ZERO -> ZERO
  |SUCC n -> SUCC (natadd n n2)

let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO
  | SUCC ZERO -> n2
  | SUCC n1' ->
    SUCC( match n2 with
      | ZERO -> ZERO
      | SUCC ZERO -> SUCC ZERO
      | SUCC n2' -> SUCC (natmul n1' (natmul n1 n2'))
    )
```

```
Test cases :
 natmul (ZERO) (SUCC ZERO) = ZERO
 natmul (SUCC ZERO) (SUCC ZERO) = SUCC ZERO
 natmul (SUCC(SUCC ZERO)) (SUCC(SUCC(SUCC ZERO)))
  = SUCC(SUCC(SUCC(SUCC(SUCC(SUCC ZERO)))))
```

Example2: Natural Numbers

- More complicated program

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type nat =
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let rec natadd n1 n2 =
  match n1 with
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  match n1 with
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  | SUCC ZERO -> n2
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   SUCC( match n2 with
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     | SUCC ZERO -> SUCC ZERO
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```

```
Test cases :
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 natmul (SUCC(SUCC ZERO)) (SUCC(SUCC(SUCC ZERO)))
  = SUCC(SUCC(SUCC(SUCC(SUCC(SUCC ZERO)))))
```

Wrong formula:

$$2 + (n_1 - 1) \times (n_1 \times (n_2 - 1))$$

Example2: Natural Numbers

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let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO
  | SUCC ZERO -> n2
  | SUCC n1’ ->
   SUCC( match n2 with
      | ZERO -> ZERO
      | SUCC ZERO -> SUCC ZERO
      | SUCC n2’ -> SUCC (natmul n1’ (natmul n1 n2’))
   )
```

```
Test cases :
 natmul (ZERO) (SUCC ZERO) = ZERO
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 natmul (SUCC(SUCC ZERO)) (SUCC(SUCC(SUCC ZERO)))
  = SUCC(SUCC(SUCC(SUCC(SUCC(SUCC ZERO))))))
```

Wrong formula:

$$2 + (n_1 - 1) \times (n_1 \times (n_2 - 1))$$

Correct formula:

$$n_1 \times n_2 = \begin{cases} 0 & n_1 = 0 \\ n_2 + (n_1 - 1) \times n_2 & n_1 \neq 0 \end{cases}$$

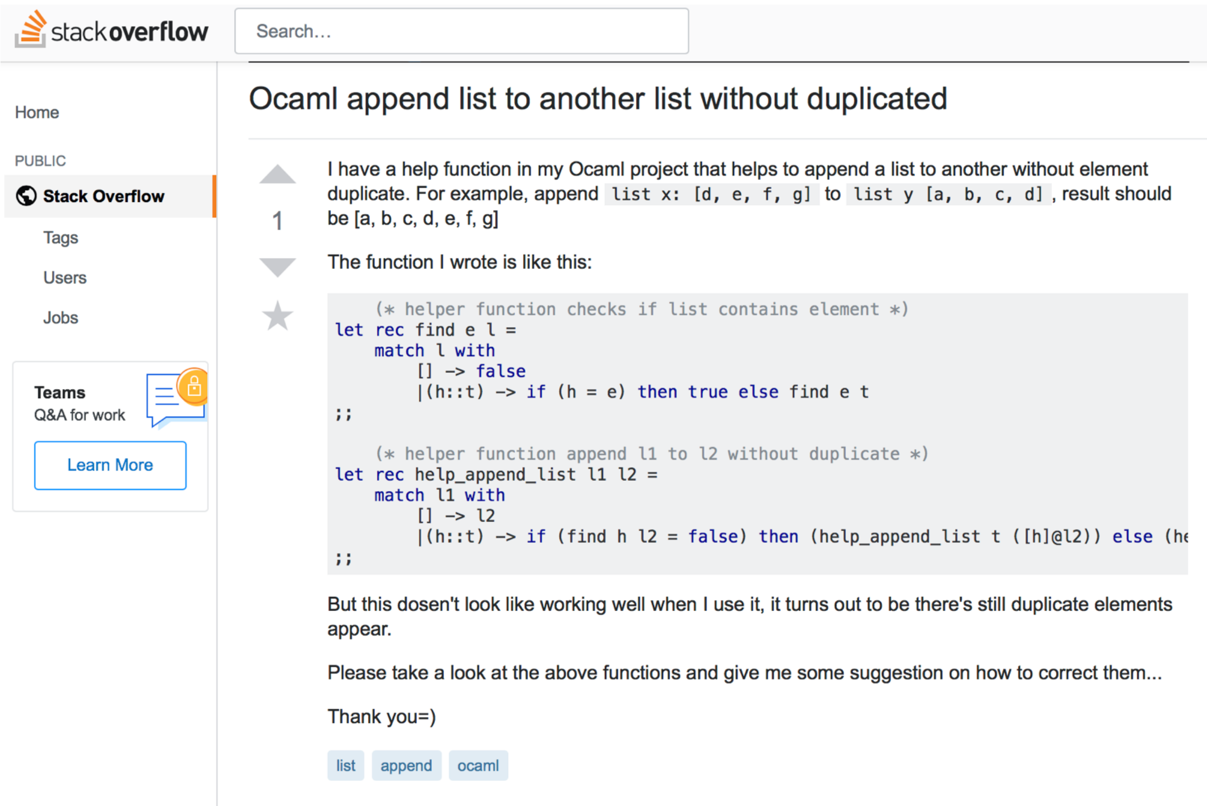
```
FixML:
natadd n2(natmul n1’ n2)
```

Time: 22 sec

Example3: Append Lists

- Stackoverflow example

```
Test cases :
append_list [1;3] [3;4;5] = [3;4;5;1]
append_list [1] [3;3;4] = [3;4;1]
```




```
(* check whether the element e is in list l *)
let rec find e l =
  match l with
  | [] -> false
  | h::t -> if h = e then true else find e t

(* append l1's elements not in l2 *)
let rec helper l1 l2 =
  match l1 with
  | [] -> l2
  | h::t ->
    if find h l2 = false then helper t (l2@[h])
    else helper t l2

let append_list x y = helper x y
```

Example3: Append Lists

- Stackoverflow example

```
Test cases :
append_list [1;3] [3;4;5] = [3;4;5;1]
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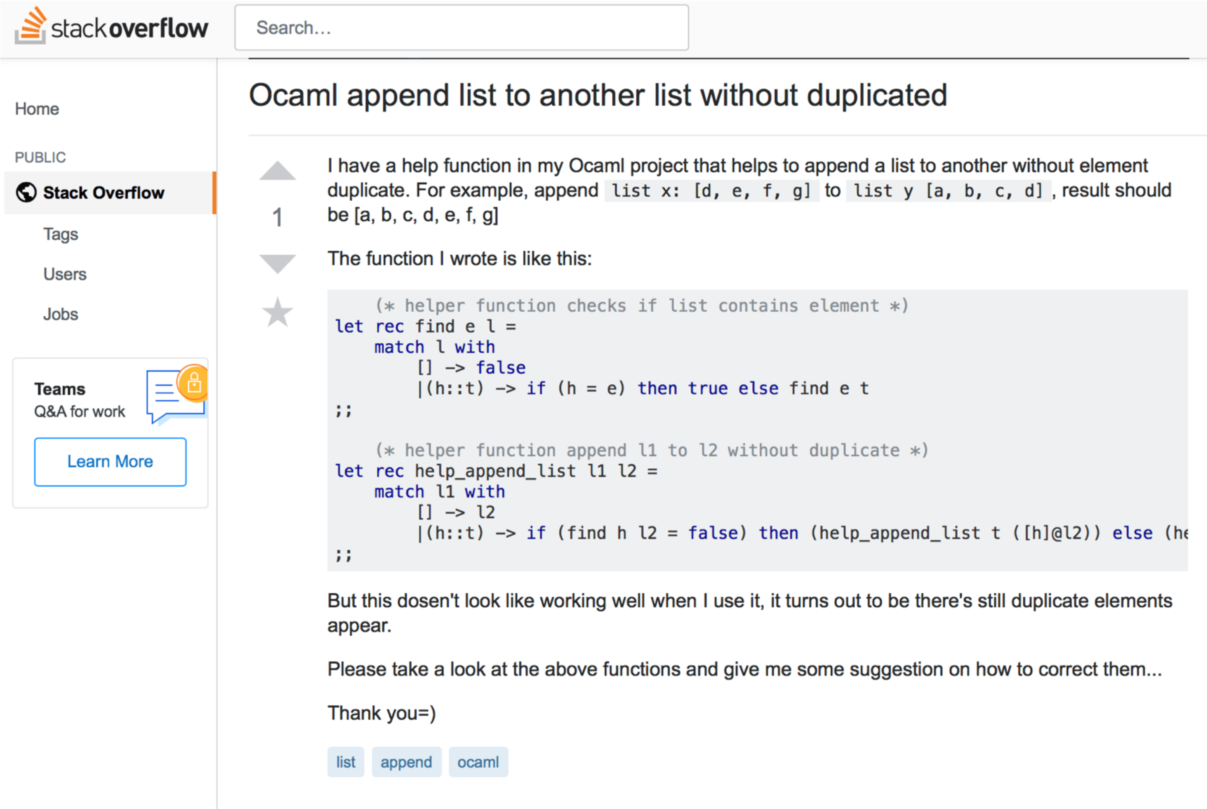


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let rec helper l1 l2 =
  match l1 with
  | [] -> l2
  | h::t ->
    if find h l2 = false then helper t (l2@[h])
    else helper t l2

let append_list x y = helper x y
```

append_list [1] [3;3;4] = [3;3;4;1]

Do not check the duplication in list y

Example3: Append Lists

- Stackoverflow example

```
Test cases :
append_list [1;3] [3;4;5] = [3;4;5;1]
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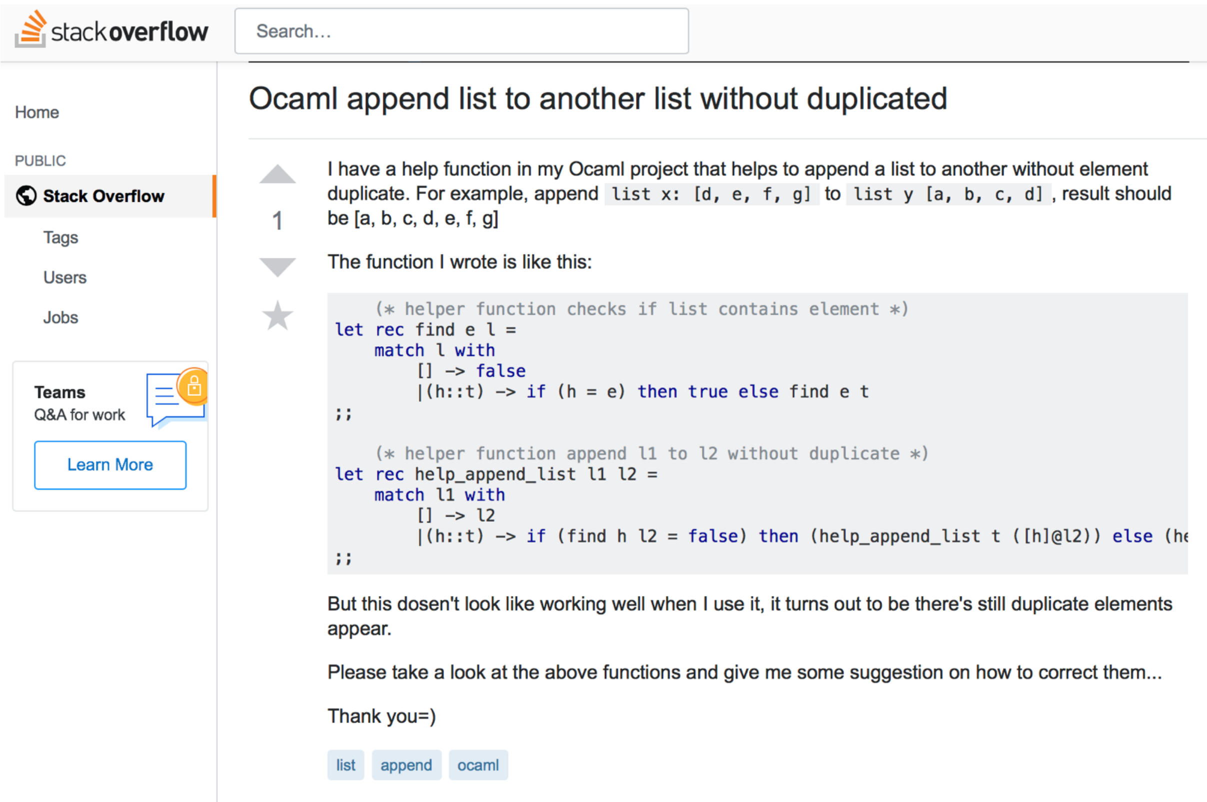
```




```
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let rec helper l1 l2 =
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  | [] -> l2
  | h::t ->
    if find h l2 = false then helper t (l2@[h])
    else helper t l2

let append_list x y = helper x y
```

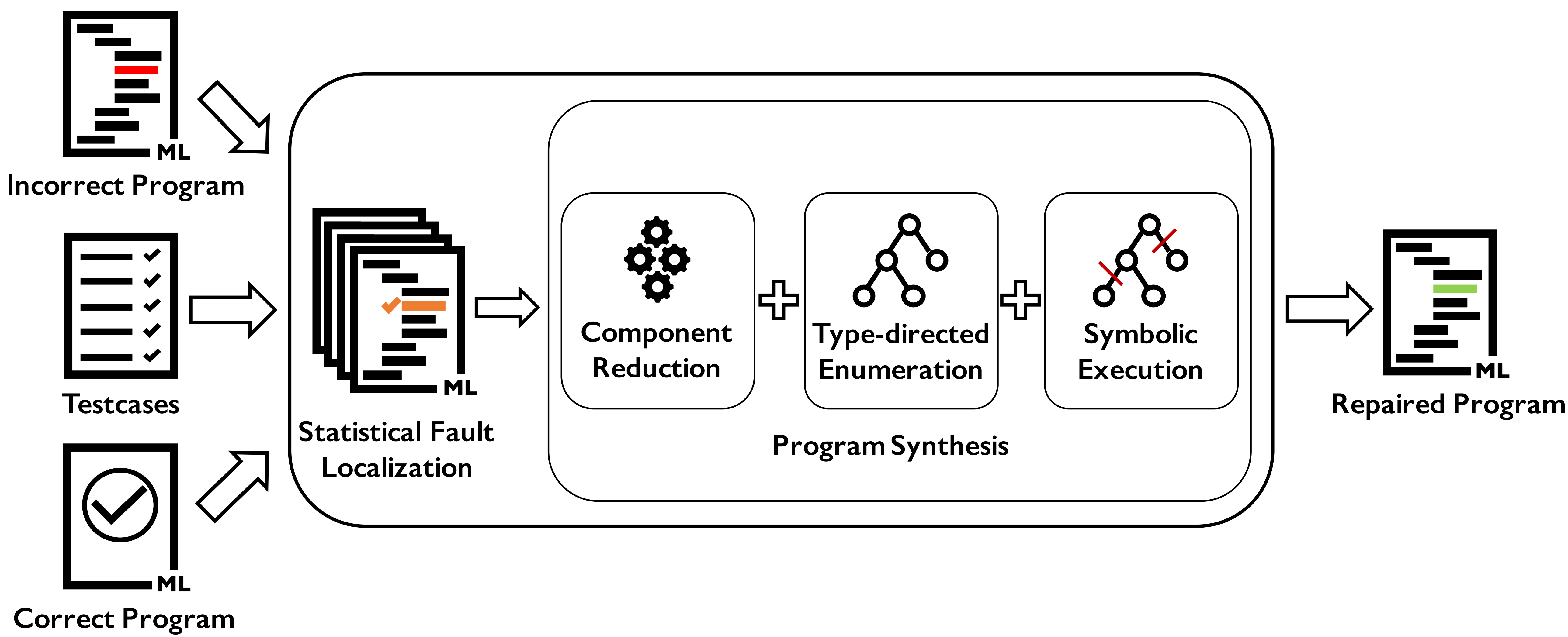
Do not check the duplication in list y

FixML: (helper y [])

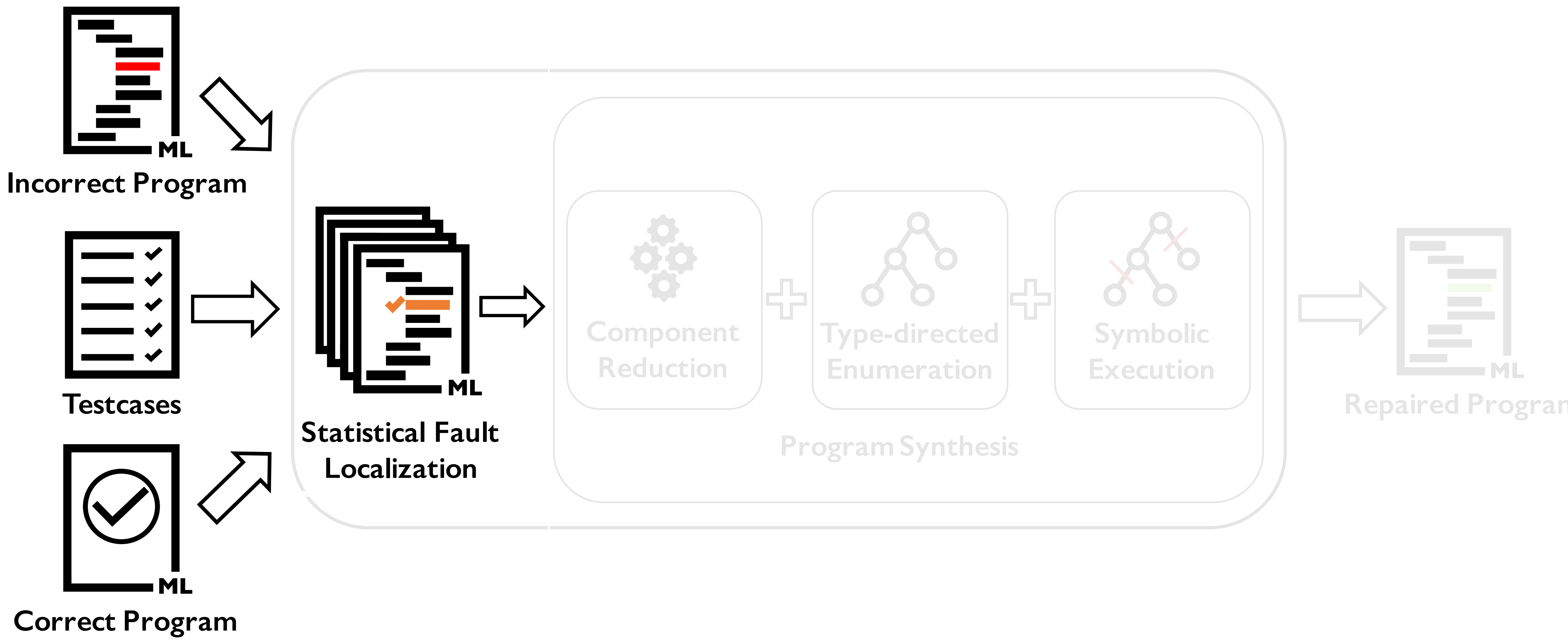
Time: 43 sec

FixML

- Given solution and test cases, our system automatically fixes the student submissions.



Error Localization



- Given buggy program and test cases, return a set of partial programs with suspicious score.

Statistical Fault Localization

Student's program:

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO
  | SUCC ZERO -> n2
  | SUCC n1' ->
    SUCC( match n2 with
      | ZERO -> ZERO
      | SUCC ZERO -> SUCC ZERO
      | SUCC n2' -> SUCC (natmul n1' (natmul n1 n2'))
    )
```

```
Test cases :
natmul ZERO (SUCC ZERO) = ZERO

natmul (SUCC ZERO) (SUCC ZERO) = (SUCC ZERO)

natmul (SUCC (SUCC ZERO)) ZERO = ZERO
```

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    )
```

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```

The program satisfies the test case => Positive

Statistical Fault Localization

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    )
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Test cases :
natmul ZERO (SUCC ZERO) = ZERO

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natmul (SUCC (SUCC ZERO)) ZERO = ZERO
```

The program cannot satisfy the test case => Negative

Statistical Fault Localization

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      | SUCC ZERO -> SUCC ZERO
      | SUCC n2' -> SUCC (natmul n1' (natmul n1 n2'))
    )
```

```
Test cases :
natmul ZERO (SUCC ZERO) = ZERO

natmul (SUCC ZERO) (SUCC ZERO) = (SUCC ZERO)

natmul (SUCC (SUCC ZERO)) ZERO = ZERO
```

- Only positive
- Positive + negative
- Only negative

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     | SUCC n2' -> SUCC (natmul n1' (natmul n1 n2'))
  )
```

```
Test cases :
natmul ZERO (SUCC ZERO) = ZERO

natmul (SUCC ZERO) (SUCC ZERO) = (SUCC ZERO)

natmul (SUCC (SUCC ZERO)) ZERO = ZERO
```

- Only positive
- Positive + negative
- Only negative

More negative, less positive => more suspicious

Statistical Fault Localization

Student’s program:

```
let rec natmul n1 n2 =
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    )
```

```
Test cases :
natmul ZERO (SUCC ZERO) = ZERO

natmul (SUCC ZERO) (SUCC ZERO) = (SUCC ZERO)

natmul (SUCC (SUCC ZERO)) ZERO = ZERO
```

Return a set of scored partial programs

P_1

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO
  | SUCC ZERO -> n2
  | SUCC n1’ -> ?
```

…

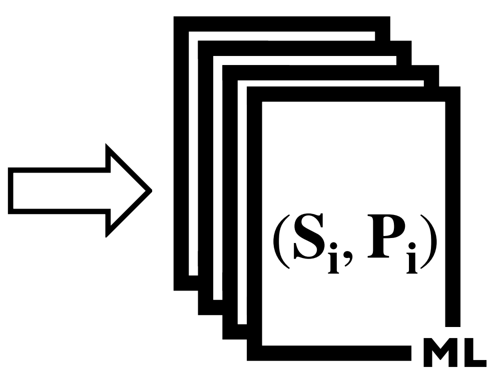
P_2

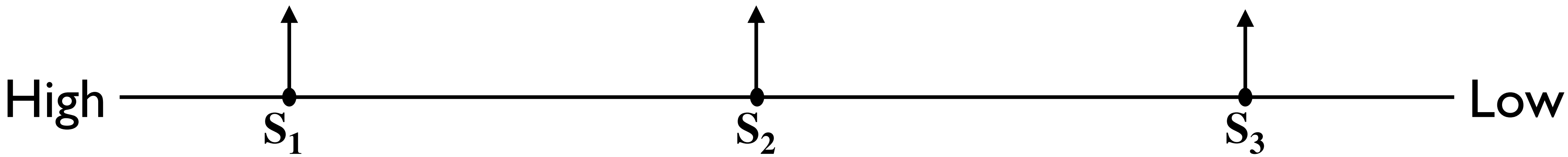
```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO
  | SUCC ZERO -> ?
  | SUCC n1’ -> …
```

…

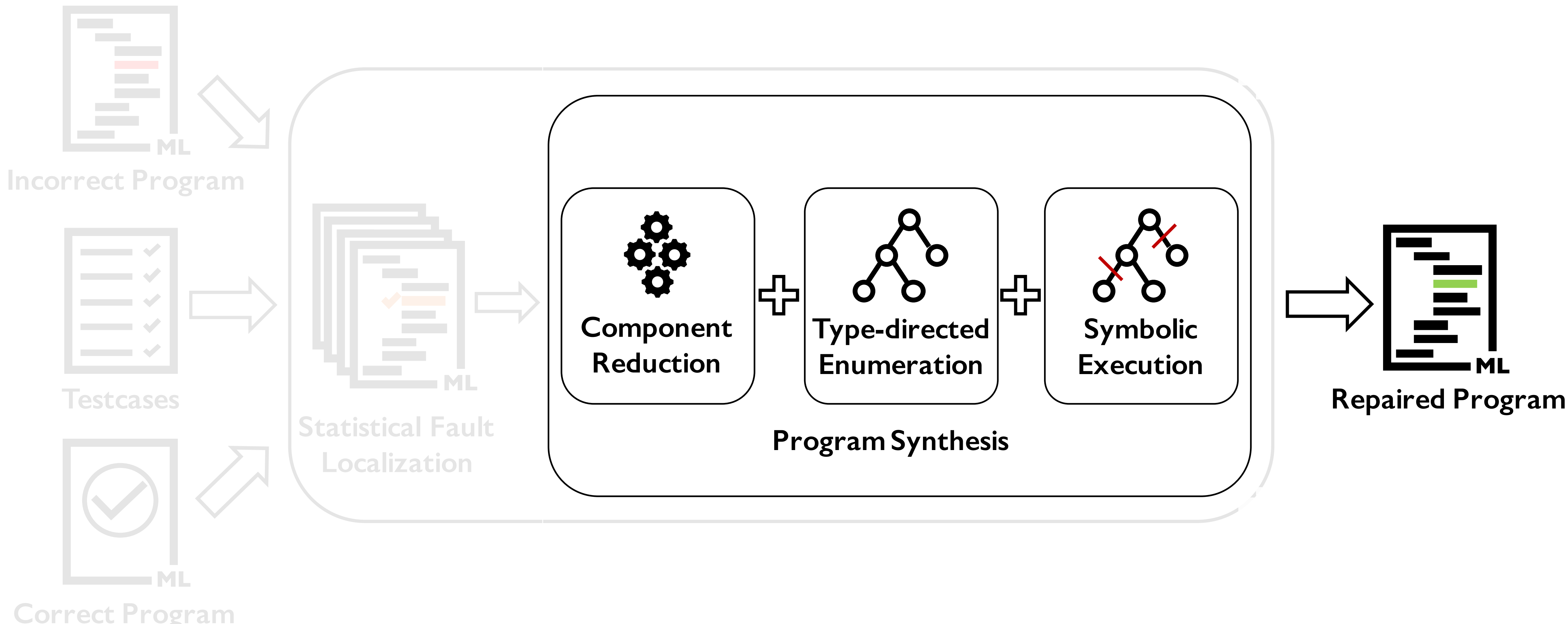
P_3

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ?
  | SUCC ZERO -> SUCC ZERO
  | SUCC n1’ -> …
```





Program Synthesis



- Given the set of scored partial program, it generates a repaired program.

Baseline: Enumerative Search

- Enumerating all expressions in the language

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO | SUCC ZERO -> n2
  | SUCC n1' -> ?
```

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO | SUCC ZERO -> n2
  | SUCC n1' -> SUCC ?
```

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO | SUCC ZERO -> n2
  | SUCC n1' -> (fun x -> ?)
```

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO | SUCC ZERO -> n2
  | SUCC n1' -> n1
```

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO | SUCC ZERO -> n2
  | SUCC n1' -> if ? then ? else ?
```

...

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO | SUCC ZERO -> SUCC ZERO
  | SUCC n1' -> SUCC (ZERO)
```

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO | SUCC ZERO -> SUCC ZERO
  | SUCC n1' -> SUCC (n1')
```

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO | SUCC ZERO -> SUCC ZERO
  | SUCC n1' -> SUCC (if ? then ? else ?)
```

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO | SUCC ZERO -> SUCC ZERO
  | SUCC n1' -> SUCC (? ?)
```

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO | SUCC ZERO -> SUCC ZERO
  | SUCC n1' -> SUCC (true)
```

...

Baseline: Enumerative Search

- Enumerating all expressions in the language

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO | SUCC ZERO -> n2
  | SUCC n1' -> ?
```

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO | SUCC ZERO -> n2
  | SUCC n1' -> SUCC ?
```

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO | SUCC ZERO -> n2
  | SUCC n1' -> (fun x -> ?)
```

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO | SUCC ZERO -> n2
  | SUCC n1' -> n1
```

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO | SUCC ZERO -> n2
  | SUCC n1' -> if ? then ? else ?
```

...

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO | SUCC ZERO -> SUCC ZERO
  | SUCC n1' -> SUCC (ZERO)
```

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO | SUCC ZERO -> SUCC ZERO
  | SUCC n1' -> SUCC (n1')
```

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO | SUCC ZERO -> SUCC ZERO
  | SUCC n1' -> SUCC (if ? then ? else ?)
```

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO | SUCC ZERO -> SUCC ZERO
  | SUCC n1' -> SUCC (? ?)
```

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO | SUCC ZERO -> SUCC ZERO
  | SUCC n1' -> SUCC (true)
```

...

Extremely inefficient!

State-of-the-art: Type-directed Search

- Searching only well-typed program

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO | SUCC ZERO -> n2
  | SUCC n1' -> ?
```

Hole type : nat

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO | SUCC ZERO -> n2
  | SUCC n1' -> SUCC ?
```

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO | SUCC ZERO -> n2
  | SUCC n1' -> (fun x -> ?)
```

Expression type : t' -> t'

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO | SUCC ZERO -> n2
  | SUCC n1' -> n1
```

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO | SUCC ZERO -> n2
  | SUCC n1' -> if ? then ? else ?
```

State-of-the-art: Type-directed Search

- Searching only well-typed program

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO | SUCC ZERO -> n2
  | SUCC n1' -> ?
```

Hole type : nat

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO | SUCC ZERO -> n2
  | SUCC n1' -> SUCC ?
```

✗

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO | SUCC ZERO -> n2
  | SUCC n1' -> (fun x -> ?)
```

Expression type : t' -> t'

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO | SUCC ZERO -> n2
  | SUCC n1' -> n1
```

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO | SUCC ZERO -> n2
  | SUCC n1' -> if ? then ? else ?
```

Still inefficient in our cases!

Our Solution

- Component reduction
  - Syntactic component reduction
  - Variable component reduction
- Pruning with symbolic execution

Technique 1: Syntactic Component Reduction

- Enumerating all expressions is very expensive.

Partial Program:

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO
  | SUCC ZERO -> n2
  | SUCC n1' -> ?
```

Language:

36 expressions

$$
\begin{array}{lcl}
E & ::= & () \mid n \mid x \mid \text{true} \mid \text{false} \mid \text{str} \mid \lambda x.E \mid E_1 + E_2 \mid E_1 - E_2 \mid E_1 \times E_2 \mid E_1 / E_2 \mid E_1 \text{ mod } E_2 \mid -E \\
& \mid & \text{not } E \mid E_1 \mid\mid E_2 \mid E_1 \ \&\& E_2 \mid E_1 < E_2 \mid E_1 > E_2 \mid E_1 \leq E_2 \mid E_1 \geq E_2 \mid E_1 = E_2 \mid E_1 <> E_2 \\
& \mid & E_1 \ E_2 \mid E_1 :: E_2 \mid E_1 @ E_2 \mid E_1 \hat{} E_2 \mid \text{raise } E \mid (E_1, \ldots, E_k) \mid [E_1; \ldots; E_k] \\
& \mid & \text{if } E_1 \ E_2 \ E_3 \mid c(E_1, \ldots, E_k) \mid \text{let } x = E_1 \text{ in } E_2 \mid \text{let rec } f(x) = E_1 \text{ in } E_2 \\
& \mid & \text{let } x_1 = E_1 \text{ and} \ldots \text{and } x_k = E_k \text{ in } E \mid \text{let rec } f_1(x_1) = E_1 \text{ and} \ldots \text{and } f_k(x_k) = E_k \text{ in } E \\
& \mid & \text{match } E \text{ with } p_1 \to E_1 \mid \cdots \mid p_k \to E_k \\
& \mid & \square
\end{array}
$$

Solution:

```
let rec natmul n1 n2 =
  match n1 with
  |ZERO -> ZERO
  | SUCC n1' -> natadd n2 (natmul n1' n2)
```

Technique 1: Syntactic Component Reduction

- Enumerating all expressions is very expensive.

Partial Program:

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO
  | SUCC ZERO -> n2
  | SUCC n1' -> ?
```

Language:

36 expressions

$$
\begin{array}{lll}
E & ::= & () \mid n \mid x \mid \text{true} \mid \text{false} \mid \text{str} \mid \lambda x.E \mid E_1 + E_2 \mid E_1 - E_2 \mid E_1 \times E_2 \mid E_1/E_2 \mid E_1 \text{ mod } E_2 \mid -E \\
& \mid & \text{not } E \mid E_1 \mid\mid E_2 \mid E_1 \;\&\&E_2 \mid E_1 < E_2 \mid E_1 > E_2 \mid E_1 \leq E_2 \mid E_1 \geq E_2 \mid E_1 = E_2 \mid E_1 <> E_2 \\
& \mid & E_1\ E_2 \mid E_1 :: E_2 \mid E_1 @ E_2 \mid E_1 \hat{} E_2 \mid \text{raise } E \mid (E_1, \ldots, E_k) \mid [E_1; \ldots; E_k] \\
& \mid & \text{if } E_1\ E_2\ E_3 \mid c(E_1, \ldots, E_k) \mid \text{let } x = E_1 \text{ in } E_2 \mid \text{let rec } f(x) = E_1 \text{ in } E_2 \\
& \mid & \text{let } x_1 = E_1 \text{ and} \ldots \text{and } x_k = E_k \text{ in } E \mid \text{let rec } f_1(x_1) = E_1 \text{ and} \ldots \text{and } f_k(x_k) = E_k \text{ in } E \\
& \mid & \text{match } E \text{ with } p_1 \rightarrow E_1 \mid \cdots \mid p_k \rightarrow E_k \\
& \mid & \square
\end{array}
$$

Solution:

```
let rec natmul n1 n2 =
  match n1 with
  |ZERO -> ZERO
  | SUCC n1' -> natadd n2 (natmul n1' n2)
```

Observation:
Although the implementations are very different, used components are similar.

Technique 1: Syntactic Component Reduction

- Enumerating all expressions is very expensive.

Partial Program:

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO
  | SUCC ZERO -> n2
  | SUCC n1' -> ?
```

Language:

~~36~~ 4 expressions

$$
\begin{array}{lll}
E & ::= & () \mid n \mid x \mid \text{true} \mid \text{false} \mid \text{str} \mid \lambda x.E \mid E_1 + E_2 \mid E_1 - E_2 \mid E_1 \times E_2 \mid E_1/E_2 \mid E_1 \text{ mod } E_2 \mid -E \\
& \mid & \text{not } E \mid E_1 \mid\mid E_2 \mid E_1 \&\& E_2 \mid E_1 < E_2 \mid E_1 > E_2 \mid E_1 \leq E_2 \mid E_1 \geq E_2 \mid E_1 = E_2 \mid E_1 <> E_2 \\
& \mid & E_1\ E_2 \mid E_1 :: E_2 \mid E_1 @ E_2 \mid E_1 \hat{} E_2 \mid \text{raise } E \mid (E_1, \ldots, E_k) \mid [E_1; \ldots; E_k] \\
& \mid & \text{if } E_1\ E_2\ E_3 \mid c(E_1, \ldots, E_k) \mid \text{let } x = E_1 \text{ in } E_2 \mid \text{let rec } f(x) = E_1 \text{ in } E_2 \\
& \mid & \text{let } x_1 = E_1 \text{ and} \ldots \text{and } x_k = E_k \text{ in } E \mid \text{let rec } f_1(x_1) = E_1 \text{ and} \ldots \text{and } f_k(x_k) = E_k \text{ in } E \\
& \mid & \text{match } E \text{ with } p_1 \rightarrow E_1 \mid \cdots \mid p_k \rightarrow E_k \\
& \mid & \square
\end{array}
$$

Solution:

```
let rec natmul n1 n2 =
  match n1 with
  |ZERO -> ZERO
  | SUCC n1' -> natadd n2 (natmul n1' n2)
```

Enumerating expressions only used in solution

Technique 2: Variable Component Reduction

- Enumerating all variables generates redundant programs.

Partial Program:

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO
  | SUCC ZERO -> n2
  | SUCC n1' -> ?
```

Bound Variable: {natmul, n1, n2, n1'}

Enumeration

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO
  | SUCC ZERO -> n2
  | SUCC n1' -> SUCC n1'
```

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO
  | SUCC ZERO -> n2
  | SUCC n1' -> n1
```

Technique 2: Variable Component Reduction

- Enumerating all variables generates redundant programs.

Partial Program:

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO
  | SUCC ZERO -> n2
  | SUCC n1' -> ?
```

Bound Variable: {natmul, n1, n2, n1'}

```
n1 = SUCC n1'
```

Enumeration

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO
  | SUCC ZERO -> n2
  | SUCC n1' -> SUCC n1'
```

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO
  | SUCC ZERO -> n2
  | SUCC n1' -> n1
```

Semantically equivalent programs

Technique 2: Variable Component Reduction

- Enumerating all variables generates redundant programs.

Partial Program:

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO
  | SUCC ZERO -> n2
  | SUCC n1' -> ?
```

Bound Variable: {natmul, ~~n1~~, n2, n1'}

```
n1 = SUCC n1'
```

Data-flow analysis:
n1 can be always expressed with n1'

Enumeration

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO
  | SUCC ZERO -> n2
  | SUCC n1' -> SUCC n1'
```

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO
  | SUCC ZERO -> n2
  | SUCC n1' -> n1
```

Choosing the minimal set of variables through data-flow analysis

Technique 3: Symbolic Execution

- Programs eventually inconsistent with the test cases

Partial Program:

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO
  | SUCC ZERO -> n2
  | SUCC n1' -> SUCC ?
```

```
Test cases :
natmul ZERO (SUCC ZERO) = ZERO

natmul (SUCC ZERO) (SUCC ZERO) = (SUCC ZERO)

natmul (SUCC (SUCC (ZERO))) ZERO = ZERO
```

Technique 3: Symbolic Execution

- Programs eventually inconsistent with the test cases

Partial Program:

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO
  | SUCC ZERO -> n2
  | SUCC n1' -> SUCC ?
```

```
Test cases :
natmul ZERO (SUCC ZERO) = ZERO

natmul (SUCC ZERO) (SUCC ZERO) = (SUCC ZERO)

natmul (SUCC (SUCC (ZERO))) ZERO = ZERO
```

Symbolic execution:

```
natmul (SUCC (SUCC (ZERO))) ZERO => (SUCC ?)
```

Technique 3: Symbolic Execution

- Programs eventually inconsistent with the test cases

Partial Program:

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO
  | SUCC ZERO -> n2
  | SUCC n1' -> SUCC ?
```

```
Test cases :
natmul ZERO (SUCC ZERO) = ZERO

natmul (SUCC ZERO) (SUCC ZERO) = (SUCC ZERO)

natmul (SUCC (SUCC (ZERO))) ZERO = ZERO
```

Symbolic execution:

```
natmul (SUCC (SUCC (ZERO))) ZERO => (SUCC ?)
```

SAT (SUCC ? = ZERO) => UNSAT

Technique 3: Symbolic Execution

- Programs eventually inconsistent with the test cases

Partial Program:

```
let rec natmul n1 n2 =
  match n1 with
  | ZERO -> ZERO
  | SUCC ZERO -> n2
  | SUCC n1' -> SUCC ?
```

```
Test cases :
natmul ZERO (SUCC ZERO) = ZERO

natmul (SUCC ZERO) (SUCC ZERO) = (SUCC ZERO)

natmul (SUCC (SUCC (ZERO))) ZERO = ZERO
```

Symbolic execution:

```
natmul (SUCC (SUCC (ZERO))) ZERO => (SUCC ?)
```

SAT (SUCC ? = ZERO) => UNSAT

Safely pruning the partial programs

Evaluation

- Evaluated on 497 programs written in OCaml with logical errors from 13 assignments

- Various task from introductory to advanced (2-154 lines) problems

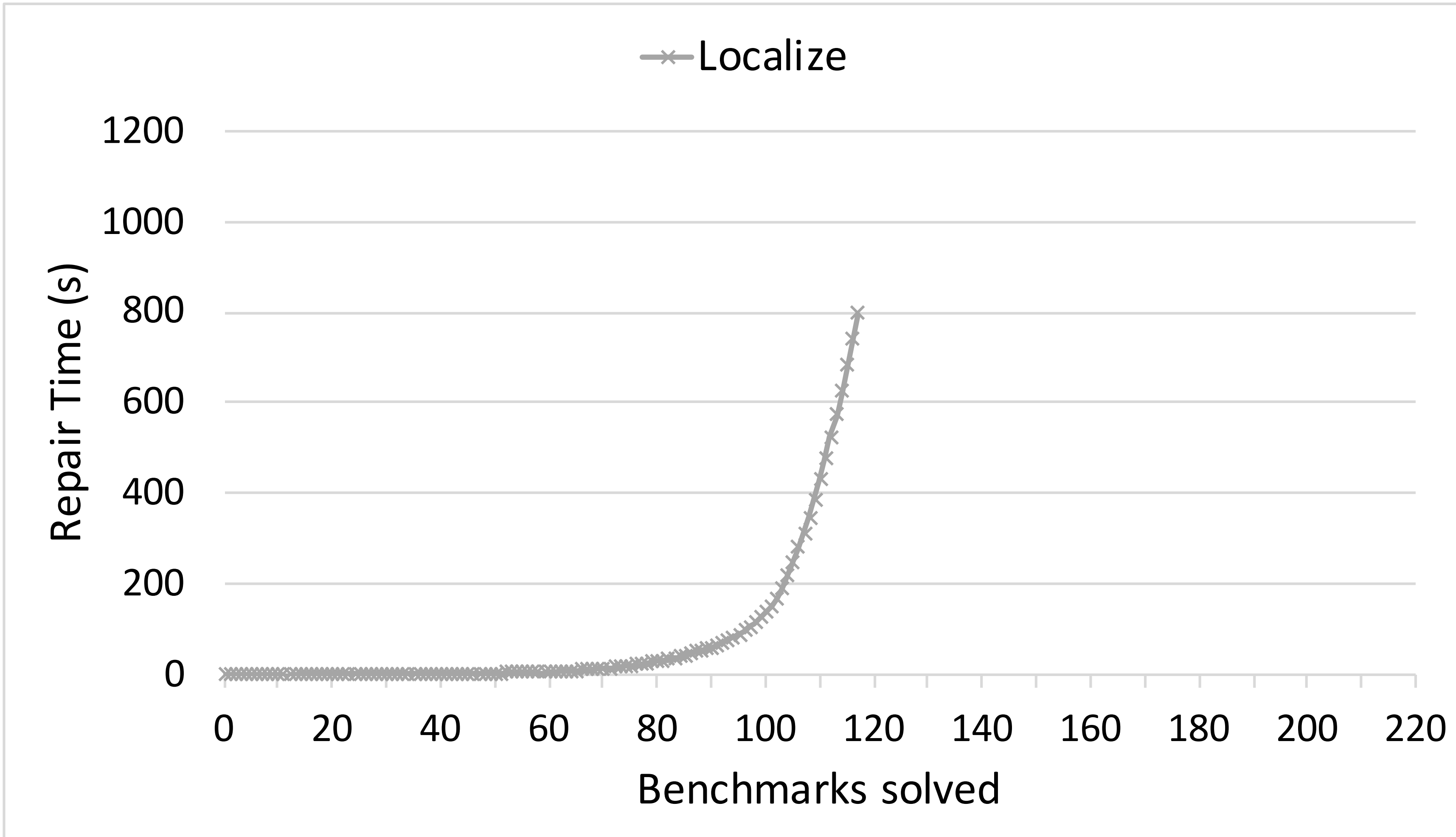
- Conducted user study with 18 students.

Effectiveness

No	Problem Description	#P	#T	LOC (min-max)	Time	Fix Rate (#Fix)	
1	Filtering elements satisfying a predicate in a list	3	10	6 (6-7)	13.0	100% (3)	Introductory Fix: 89% Time: 2.5 sec
2	Finding a maximum element in a list	32	10	8 (4-14)	0.2	100% (32)	
3	Mirroring a binary tree	9	10	11 (9-14)	0.1	89% (8)	
4	Checking membership in a binary tree	15	17	11 (9-18)	5.2	80% (12)	
5	Computing $\sum_{i=j}^{k} f(i)$ for j, k, and f	23	11	5 (2-9)	4.2	78% (18)	
6	Adding and multiplying user-defined natural numbers	34	10	20 (13-50)	20.6	59% (20)	Intermediate Fix: 48% Time: 11.6 sec
7	Finding the number of ways of coin-changes	9	10	21 (6-35)	2.6	44% (4)	
8	Composing functions	28	12	7 (3-19)	5.5	43% (12)	
9	Implementing a leftist heap using a priority queue	20	13	43 (33-72)	2.6	40% (8)	
10	Evaluating expressions and propositional formulas	101	17	32 (17-57)	1.2	39% (39)	Advanced Fix: 30% Time: 4.8 sec
11	Adding numbers in user-defined number system	14	10	52 (19-138)	7.0	36% (5)	
12	Deciding lambda terms are well-formed or not	86	11	30 (13-79)	1.3	26% (22)	
13	Differentiating algebraic expressions	123	17	36 (14-154)	11.4	25% (31)	
	Total / Average	497	158	27 (2-154)	5.4	43% (214)	

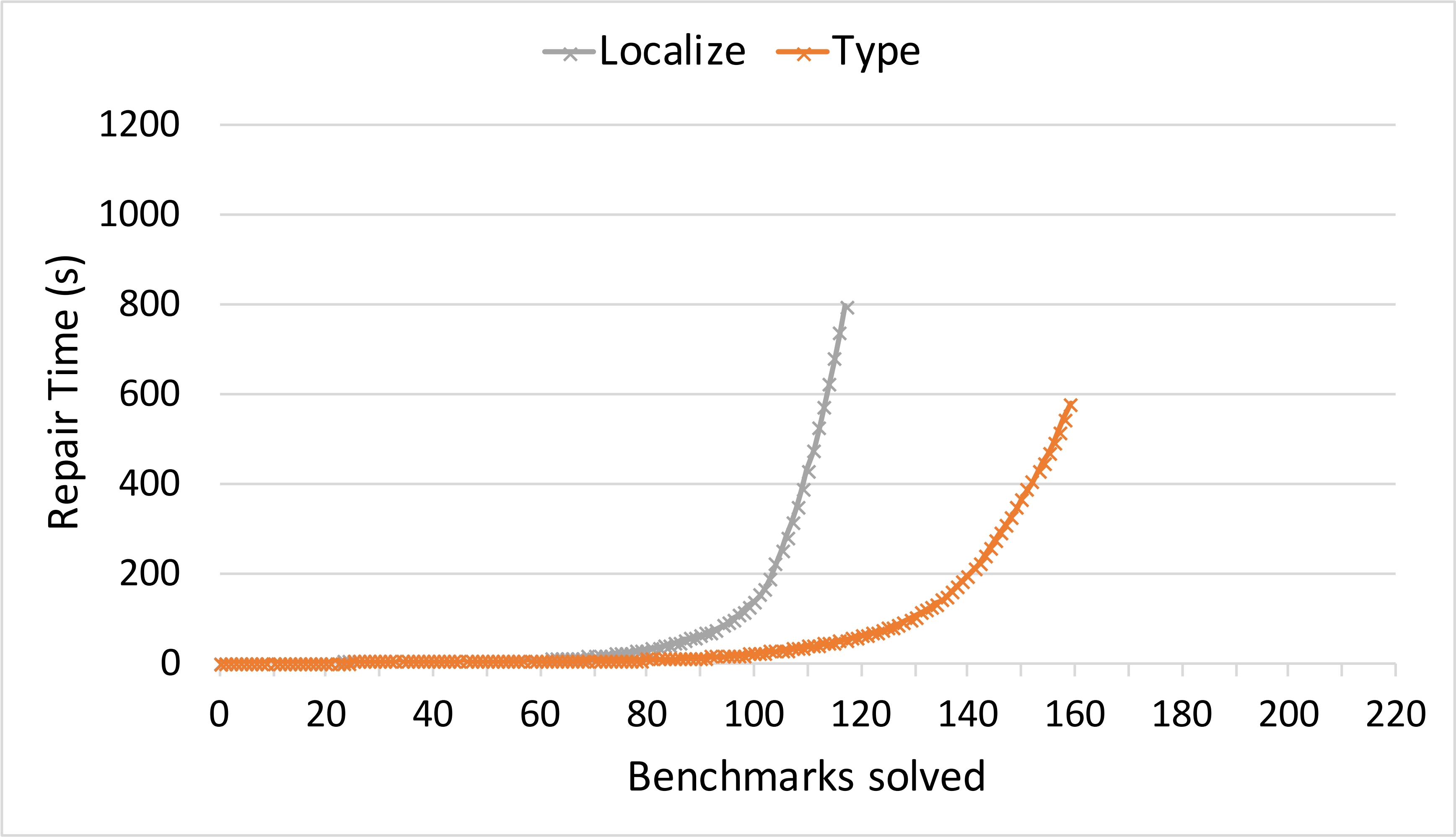
- Average time: 5.4 sec / Fix rate: 43%
- Generating patches for diverse problems

Technique Utility



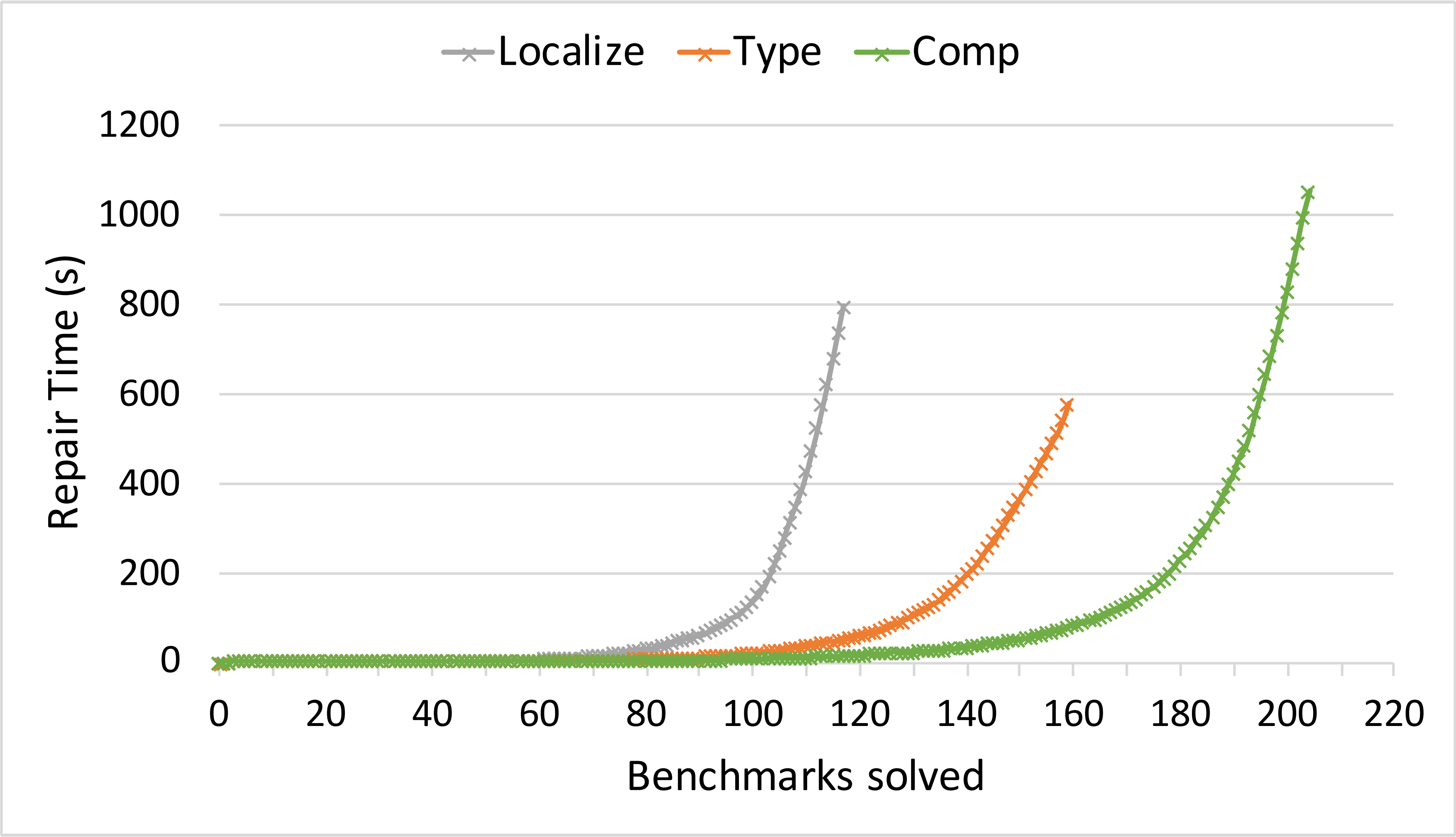
- Only statistical fault localization with enumerative search

Technique Utility



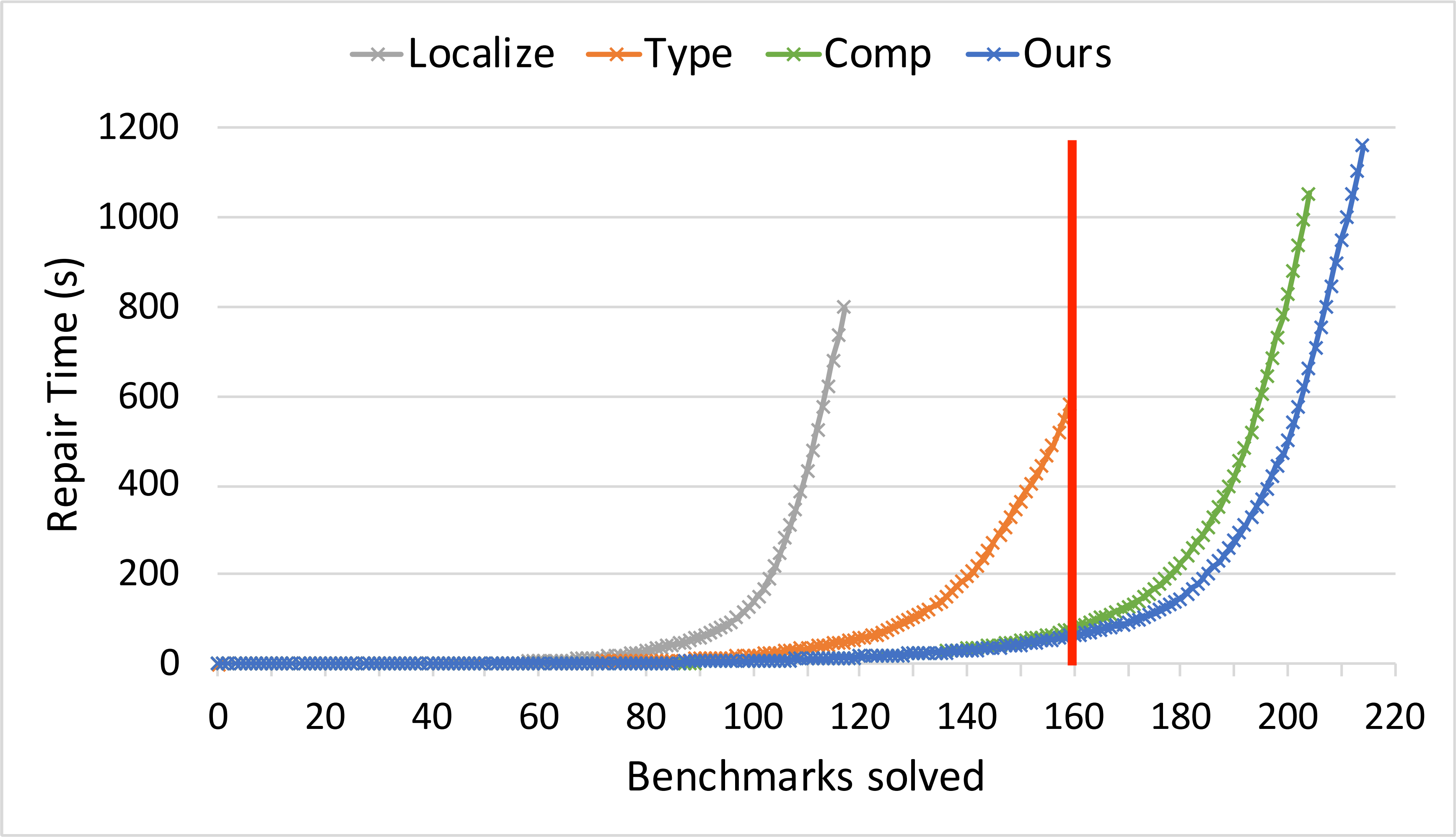
- Statistical fault localization + type-directed search

Technique Utility



- Localization + type-directed search + component reduction

Technique Utility



- Localization + type + component + symbolic execution
- Compare to Type : 579sec vs 65sec (x 8.9 faster)
  160 vs 214 (54 submissions more)

User Study

- Conducted user study with 18 undergraduate students.

- Requested to solve three problems.

- Provide feedback and survey it.

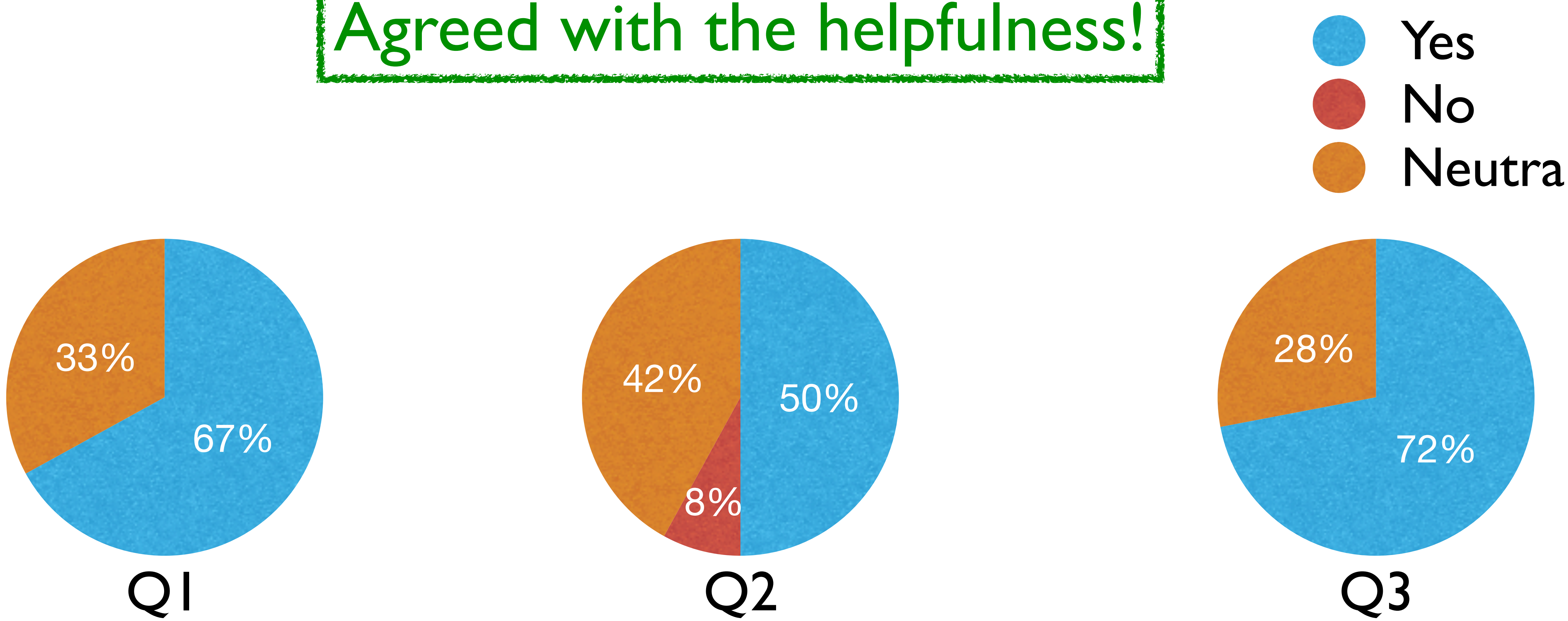
Helpfulness

Q1. Does the tool generate better corrections?

Q2. Does the feedback help to understand your mistakes?

Q3. Is the tool overall useful in learning functional programming?

Agreed with the helpfulness!



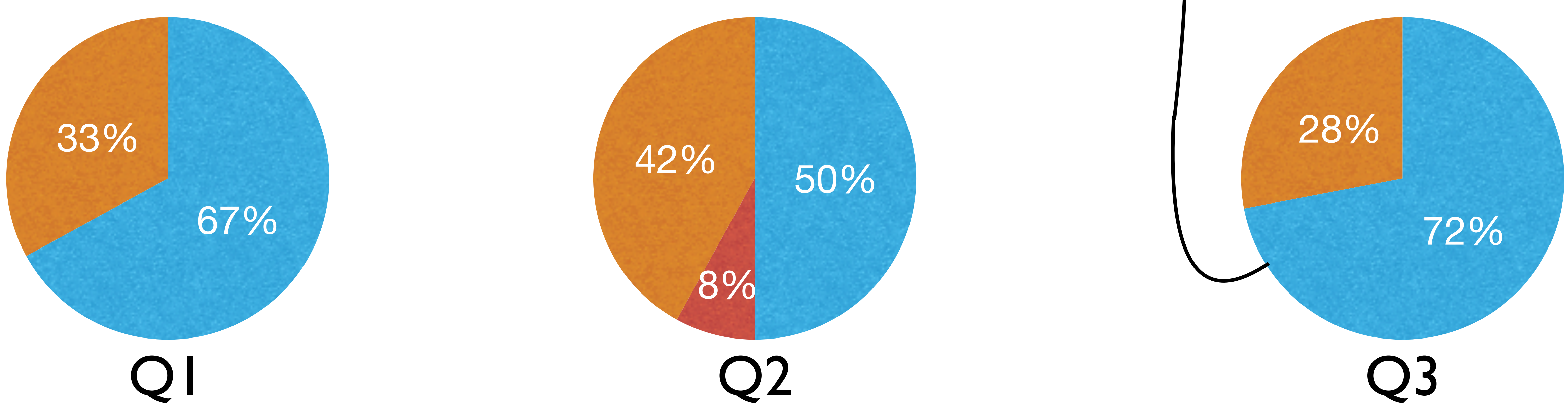
Helpfulness

Q1. Does the tool generate better corrections?

Q2. Does the feedback help to understand your mistakes?

Q3. Is the tool overall useful in learning functional programming?

"Since the tool generates the optimal patch, we can learn the programming skills from it"



Limitations

- Multiple Errors

Buggy implementation:

```
let rec eval f =
  match f with
  | True -> true | False -> false
  | Not e - > if e = True then false else true
  | AndAlso (e1, e2) -> if e1 = True && e2 = True then true else false
  | OrElse (e1, e2) -> if e1 = False && e2 = False then false else true
  | Imply (e1, e2) -> if e1 = True && e2 = False then false else true |
```

- Dependence on test cases

Buggy implementation:

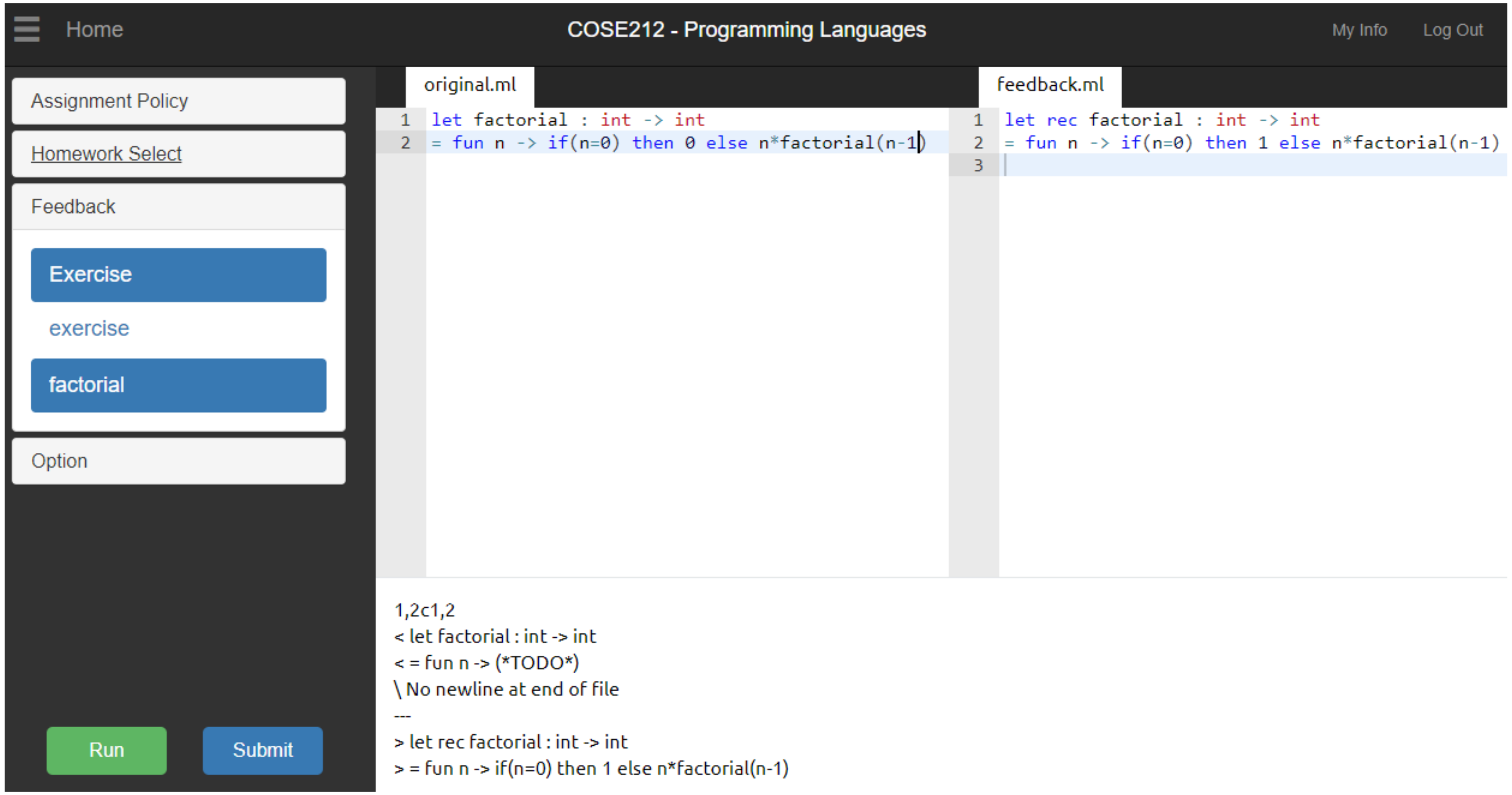
```
let rec sigma f a b =
  if f a != f b then f b + sigma f a (b-1) else f b
```

Feedback: replace (f a != f b) by (a != b)

```
Test cases :

(fun x-> x * x) 1 3 => 14

(fun x-> x + x) 1 3 => 12

(fun x-> (x*x)+x) 3 6 => 104

(fun x -> x mod 3) 1 5 => 6
```

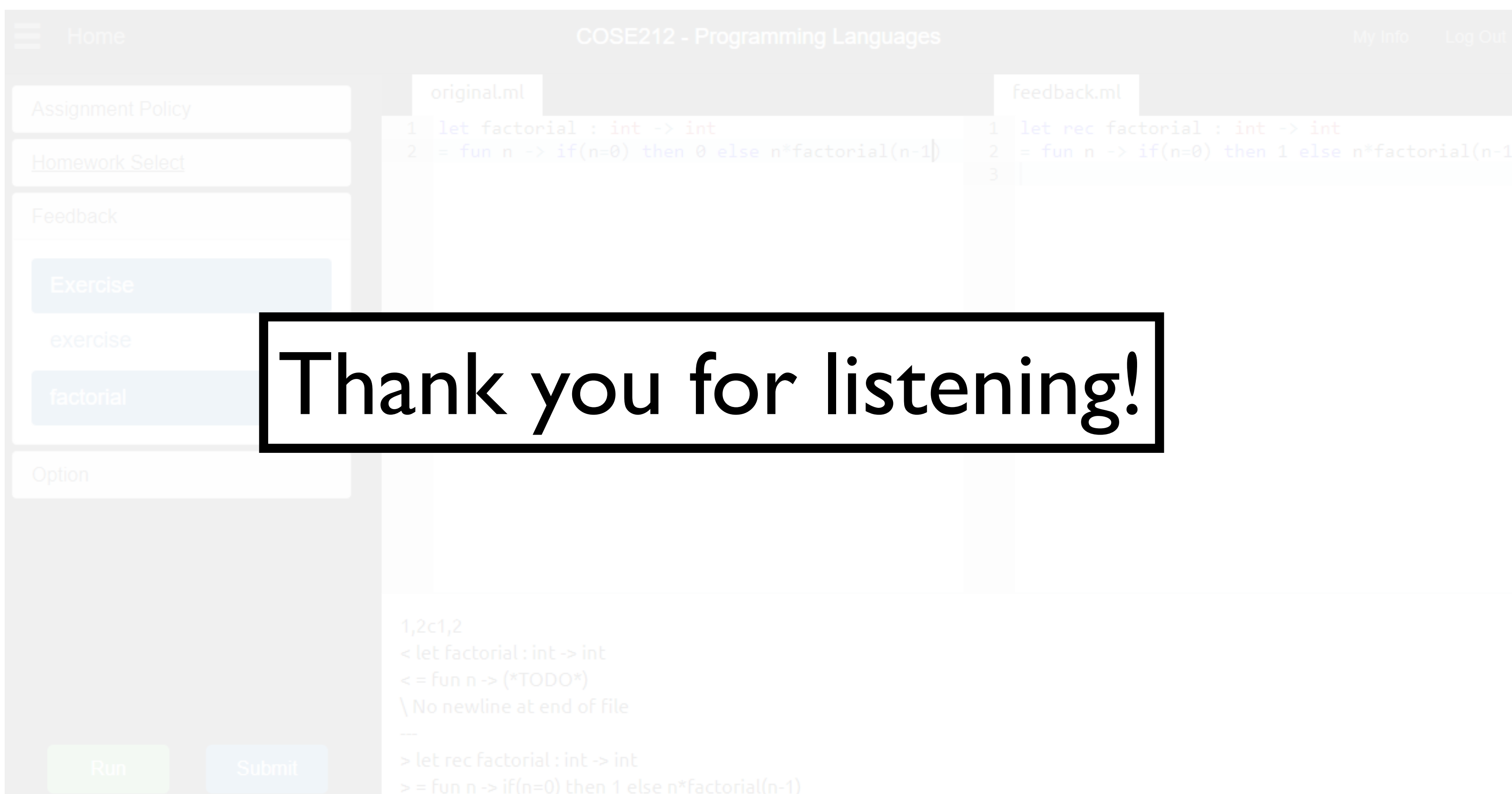
Summary

- The first system to provide personalized feedback of logical errors for functional programming assignments
- Code and our data: https://github.com/kupl/FixML
- Tool usage: https://tryml.kroea.ac.kr



Summary

- The first system providing personalized feedback of logical errors for functional programs
- Code and our data: https://github.com/kupl/FixML
- Tool usage: https://tryml.kroea.ac.kr



Thank you for listening!